UNCLASSIFIED

AD NUMBER
AD843091

LIMITATION CHANGES

TO:

Approved for public release; distribution is unlimited.

FROM:

Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; JUN 1968. Other requests shall be referred to Naval Ship Research and Development Center, Washington, DC.

AUTHORITY

NSRDC ltr 12 Oct 1972

THIS PAGE IS UNCLASSIFIED

FURTHER CALCULATIONS OF THE FLUTTER SPEED OF A FULLY SUBMERGED SUBCAVITATING HYDROFOIL

by

Wen-Hwa Chu and H. Norman Abramson

Prepared for

The U.S. Navy
Naval Ship Research and Development Center
Washington, D.C. 20007

June 1968

SOUTHWEST RESEARCH INSTITUTE
SAN ANTONIO   HOUSTON

40

FURTHER CALCULATIONS OF THE FLUTTER SPEED OF A FULLY SUBMERGED SUBCAVITATING HYDROFOIL

by

Wen-Hwa Chu and H. Norman Abramson

Technical Report
Contract No. N00014-68-C-0259
SwRI Project 02-2311

D D C

NOV 18 1968

D

Prepared for

The U.S. Navy
Naval Ship Research and Development Center
Washington, D.C. 20007

June 1968

Approved:

H. Norman Abramson, Director
Department of Mechanical Sciences

ABSTRACT

Some new calculations of the flutter speed of the
SwRI fully submerged subcavitating hydrofoil
model are presented. Variations in lift curve
slope and in center of pressure location are found
to have a most profound influence on both flutter
speed and frequency. When variations in these
parameters are combined with a relaxation of the
Kutta condition (proposed previously), excellent
agreement with the measured flutter speed is
obtained.

TABLE OF CONTENTS

INTRODUCTION

The catastrophic structural failure of the SwRI flutter model as reported by Abramson and Ransleben (Ref. 1) has been the focal point of much discussion and the subject of a variety of calculations and analyses. The conditions at which catastrophic failure of this model occurred were a velocity of 48.1 knots and a frequency of 17.5 cps (corresponding to a reduced velocity of 1.48). However, in making any comparison or correlations with these data, it should be quite clearly noted that uncertainties exist even here (Refs. 2, 3), so that exact agreement should be viewed with as much skepticism as would wide disagreement.

Initial attempts to calculate the flutter velocity of this model by two-dimensional and Reissner-Stevens theories failed to yield reasonable results (Refs. 1, 4), although the model had an aspect ratio of five. This difficulty was not unanticipated, however, as large differences between experimental data and computed results for various flutter studies at low mass density ratios had previously been encountered and had been the subject of much controversy (Refs. 2, 5). It therefore has now become rather widely accepted that, for the range of parameters pertinent to hydrofoil applications the more-or-less classical formulations of the unsteady hydrodynamic loads are not adequate. However, even the evidence available here is somewhat clouded: for example, measured data on oscillatory lift and moment distributions (Refs. 6, 7) employed directly in a flutter analysis of the SwRI flutter model yielded a substantially overconservative prediction (Refs. 1, 4); unfortunately, the accuracy of this

2

measured data is also open to some question (Ref. 3), and the analytical techniques employed in the analysis may not have been adequate.

A somewhat different approach was also taken quite early in all of these studies by considering some relaxation of the Kutta condition (Refs. 8, 9); but, again, reasonable flutter predictions were not readily forthcoming (Ref. 4) and could only be obtained from this semiempirical procedure by using rather large arbitrary shifts in the phase of the circulation function.

Several years ago, Yates began the development of a modified strip-analysis method employing arbitrary spanwise distributions of lift curve slope and center of pressure location (Refs. 10-14). This method can be based on the idea of employing measured data for these parameters or, alternatively, employing values calculated from lifting surface theory. Yates has applied this basic method to the SwRI flutter model (Ref. 15), obtaining a flutter velocity of approximately 38.5 knots, with a structural damping coefficient of $g = 0.02$ cps and a flutter frequency of 21.3 cps. Some further refinements and modifications appear to have yielded somewhat better agreement, but these have not yet been reported in detail.

Yet another approach has relied upon the rather full application of lifting surface theories (Refs. 16-18). The method proposed by Rowe (Ref. 18) provides for some variation in satisfaction of the Kutta condition and inherently generates data on lift curve slope and center pressure location.

It is the purpose of the present report to present yet another series of results. These are all based essentially upon strip theory, but involving various modifications to account for Kutta condition, lift curve slope, center of pressure location, etc.

ANALYSIS

Basic Equations

For combined pitching and heaving oscillations of a hydrofoil, represented as a uniform unswept beam, the equations of motion may be put into the nondimensional form (Ref. 19)

$$-\mu\gamma_h^2 h_0^* - \mu\gamma_{ah}x_a a_0 + \mu\gamma_h^2 r_\omega^2 \frac{\omega_a^2}{\omega^2}(1+ig)h_0^* = A_{hh}^* h_0^* + A_{ha}^* a_0 \qquad (1a)$$

$$-\mu\gamma_{ah}x_a h_0^* - \mu r_a^2 \gamma_a^2 a_0 + \gamma_a^2 \frac{\omega_a^2}{\omega^2}[1+ig]\mu r_a^2 a_0 = A_{ah}^* h_0^* + A_{aa}^* a_0 \qquad (1b)$$

where

$$\gamma_h^2 = \int_0^1 f_h^2(y^*)\,dy^* \quad , \quad \gamma_a^2 = \int_0^1 f_a^2(y^*)\,dy^* \quad , \quad \gamma_{ah} = \int_0^1 f_a(y^*)\,f_h(y^*)\,dy^*$$

$$(2a, b, c)$$

f_a, f_h are the pitching and heaving mode shapes, respectively, i.e.,

$$a = a_0 f_a(y^*) \quad \text{positive nose-up} \qquad (3a)$$

$$\frac{h}{b} = h_0^* f_h(y^*) \quad \text{positive downward} \qquad (3b)$$

$$y^* = \frac{y}{sb} \quad s = \text{aspect ratio, } b = \text{semichord} \qquad (4a)$$

$$\mu = \frac{M}{\pi\rho b^2} \quad , \quad r_a^2 = \frac{I_a}{Mb^2} \qquad (4b, c)$$

$$x_a = \frac{S_a}{Mb} \quad , \quad r_\omega^2 = \frac{\omega_h^2}{\omega_a^2} \qquad (4d, e)$$

The (nondimensional) generalized loadings are

$$A_{hh}^* = \int_0^1 L_h^* f_h^2(y^*) \, dy^* \tag{5a}$$

$$A_{ha}^* = \int_0^1 L_a^* f_a(y^*) \, f_h(y^*) \, dy^* \tag{5b}$$

$$A_{ah}^* = \int_0^1 M_h^* f_h(y^*) \, f_a(y^*) \, dy^* \tag{5c}$$

$$A_{aa}^* = \int_0^1 M_a^* f_a^2(y^*) \, dy^* \tag{5d}$$

The force L* is positive downward; the moment M* is positive nose-up.

The determinant of the coefficients must vanish at the critical (flutter) speed; in this case.

$$a_2 z^2 + a_1 z + a_0 = 0 \tag{6}$$

where

$$z = \frac{\omega_a^2}{\omega^2} (1 + ig) \tag{7}$$

and

$$a_2 = \mu^2 r_a^2 r_\omega^2 \gamma_a^2 \gamma_h^2 \tag{8a}$$

$$a_1 = -\mu r_\omega^2 \gamma_h^2 (\mu r_a^2 \gamma_a^2 + A_{aa}^*) - \gamma_a^2 \mu r_a^2 (\gamma_h^2 \mu + A_{hh}^*) \tag{8b}$$

$$a_0 = (\gamma_h^2 \mu + A_{hh}^*)(\mu r_a^2 \gamma_a^2 + A_{aa}^*) - (\mu x_a \gamma_{ah} + A_{ha}^*)(\mu x_a \gamma_{ah} + A_{ah}^*) \tag{8c}$$

The standard procedure employed for determining the flutter speed from Eq. (6) is to assume a value for the reduced frequency $k = \frac{\omega b}{V}$ and then to calculate z and $\frac{\omega_a}{\omega}$. The value of g may then be found and a plot made of V vs. g; the value of V at which $g = 0$ then gives the flutter velocity.

Case 1. Reissner-Stevens Method (Ref. 20)

Following the convention of Reference 19, the expressions for force and moment corresponding to pitch and heave are, respectively,

$$L_h^* = \frac{\dfrac{L_h'}{h^*}}{\pi \rho \omega^2 b^3} = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ik\left[C(k) + \sigma_h\right]\right\} = \overline{L}_h^* e^{ia_{Lh}} \qquad (9a)$$

$$L_a^* = \frac{\dfrac{L_a'}{a}}{\pi \rho \omega^2 b^3} = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + \left[1 + ik\left(\frac{1}{2} - a\right)\right]\left[C(k) + \sigma_a\right]\right\}$$
$$= \overline{L}_a^* e^{ia_{La}} \qquad (9b)$$

$$M_h^* = \frac{\dfrac{M_h'}{h^*}}{\pi \rho \omega^2 b^4} = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - \left(a + \frac{1}{2}\right)ik\left[C(k) + \sigma_h\right]\right\} = \overline{M}_h^* e^{ia_{Mh}} \qquad (9c)$$

$$M_a^* = \frac{\dfrac{M_a'}{a}}{\pi \rho \omega^2 b^4} = -\frac{2}{k^2}\left\{\frac{1}{2}ik\left(\frac{1}{2} - a\right) - \frac{1}{2}k^2\left(\frac{1}{8} + a^2\right) - \left(a + \frac{1}{2}\right)\right.$$
$$\left. \cdot \left[1 + ik\left(\frac{1}{2} - a\right)\right]\left[C(k) + \sigma_a\right]\right\} = \overline{M}_a^* e^{ia_{Ma}} \qquad (9d)$$

The procedure required to calculate the finite span correction factors σ_h and σ_a are given in Reference 20 (for two-dimensional theory, $\sigma_h = \sigma_a = 0$).

The lift and moment calculated for the SwRI flutter model are found to be in agreement with those obtained previously with a different computer

program (Ref. 1). The V-g curve is given in Figure 1; the flutter speed lies between two previously calculated values, all of which are based upon certain data read from curves given in Reference 20, so that the basic computer program to be employed in the calculation of the curves to follow is confirmed to be correct.

The measured mode shapes and characteristics of the SwRI flutter model, as employed in these calculations, are given in Table 1.

Case 2.   Calculation Using Experimental Data for Steady-State Center of Pressure Location

The expressions for lift and moment are now written as

$$L_h^* = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ik\left[C(k) + \sigma_h\right]\right\} \tag{10a}$$

$$L_\alpha^* = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + \left[1 + ik\left(\frac{1}{2} - a\right)\right]\left[C(k) + \sigma_\alpha\right]\right\} \tag{10b}$$

$$M_h^* = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - (a - x_{cp})\,ik\,\left[C(k) + \sigma_h\right]\right\} \tag{10c}$$

$$M_\alpha^* = -\frac{2}{k^2}\left\{\frac{1}{2}ik\left(\frac{1}{2} - a\right) - \frac{1}{2}(k^2 + a^2) - (a - x_{cp})\left[1 + ik\left(\frac{1}{2} - a\right)\right]\right.$$
$$\left. \cdot\,[C(k) + \sigma_\alpha]\right\} \tag{10d}$$

where x_{cp} is the location of the center of pressure, dependent upon the spanwise positions, or obtained from the experimental data of Reference 6. In the moment expressions, Eqs. (10c) and (10d), the term $a - x_{cp}$ has been used in place of the term $\frac{1}{2} + a$ as the lift force is to be taken as acting at x_{cp} rather than at the quarter-chord point $\left(-\frac{1}{2}b\right)$.

The V-g curve is given in Figure 2, indicating a flutter speed of approximately 67 knots (compared with the experimental value of 48 knots).

Case 3. Calculation Using Experimental Data for Oscillating Center of Pressure Location

The forces will be considered to be the same as those of the classical theory of Case 1, but the moments will be taken as

$$M_h^* = \bar{M}_h^* e^{ia_{Mh}} - \bar{M}_h^* \cos(a_{Mh} - a_{Lh}) e^{ia_{Lh}} + x_{cp_h}^* L_h^* e^{ia_{Lh}} \qquad (11a)\dagger$$

$$M_a^* = \bar{M}_a^* e^{ia_{Ma}} - \bar{M}_a^* \cos(a_{Ma} - a_{La}) e^{ia_{Lh}} + x_{cp_a}^* L_a^* e^{ia_{Lh}} \qquad (11b)$$

The unsteady center of pressure locations are taken as $x_{cp_h}^*$, $x_{cp_a}^*$ in these expressions and defined (Ref. 6) so that

$$L_a^* x_{cp_a}^* = \bar{M}_a^* \cos(a_{Mh} - a_{Lh}) \qquad (12a)$$

$$L_h^* x_{cp_h}^* = \bar{M}_h^* \cos(a_{Ma} - a_{La}) \qquad (12b)$$

where

$$x_{cp_h}^* = 2\tilde{x}_{cp_h} - \frac{1}{2} - d \qquad (13a)$$

$$x_{cp_a}^* = 2\tilde{x}_{cp_a} - \frac{1}{2} - d \qquad (13b)$$

and $\tilde{x}_{cp_h}$, $\tilde{x}_{cp_a}$ are the oscillatory center of pressure locations corresponding to heave and pitch motions, respectively, in fractions of chord measured from the leading edge; d is the distance between the moment axis and the quarter-chord point. For the SwRI flutter model

---

$\dagger M_h^*$, M_a^*, a_{Mh}, a_{Ma}, L_h^*, L_a^*, a_{Lh}, a_{La} are given by Eqs. (9a-d) with $\sigma_h = \sigma_a = 0$.

$$\frac{1}{2} + d = 0.31 \tag{14}$$

Values of $x^*_{cp_h}$, $x^*_{cp_a}$ are given in Table 2.

The V-g curve is given in Figure 3, indicating a flutter speed of approximately 70 knots.

Case 4. Calculation Using Modified Values of Lift Curve Slope and Center of Pressure

It is well known, in steady flow, that the actual lift curve slope of a foil is somewhat less than the theoretical value of 2π, thus having the effect of reducing the value of the circulation. Denoting the ratio of actual to theoretical lift curve slopes by β, the Theodorsen function may be multiplied by this ratio (no additional corrections are to be applied to the Reissner-Stevens three-dimensional factor). Then, considering the lift force to act at the center of pressure, the expressions for lift and moment become

$$L_h^* = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ik\,[\beta C(k) + \sigma_h]\right\} \tag{15a}$$

$$L_a^* = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + \left[1 + ik\left(\frac{1}{2} - a\right)\right]\,[\beta C(k) + \sigma_a]\right\} \tag{15b}$$

$$M_h^* = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - (a - x_{cp})\,ik\,[\beta C(k) + \sigma_h]\right\} \tag{15c}$$

$$M_a^* = -\frac{2}{k^2}\left\{\frac{1}{2}ik\left(\frac{1}{2} - a\right) - \frac{1}{2}k^2\left(\frac{1}{8} + a^2\right) - (a - x_{cp})\left[1 + ik\left(\frac{1}{2} - a\right)\right]\right.$$
$$\left. \cdot\,[\beta C(k) + \sigma_a]\right\} \tag{15d}$$

Various values of C_{l_α} and x_{cp} are given in Table 3. The data in Table 3a are derived from experiments (Ref. 6) while those in Table 3b are derived from lifting surface theory (Ref. 15).

V-ξ curves calculated by using these two sets of C_{l_α} and x_{cp} values are given in Figures 4a and 4b. The first, employing measured foil data, gives a flutter speed of approximately 100 knots while the second, employing calculated foil data, gives a flutter velocity of approximately 76 knots. Comparing these results directly with those of Case 2, it would mean that the effect of modifying the lift curve slope in this manner is to raise the flutter velocity.

Case 5. Calculation Using Yates' Modified Strip Theory (Ref. 10)

According to Yates (Ref. 10), the constant factor $\frac{1}{2}$ - a in the downwash terms of a conventional strip theory analysis could be replaced by

$$\lambda = \beta + x_{cp} - a \tag{16}$$

which varies along the span of the foil. Since this modification includes three-dimensional effects, σ_α and σ_h are to be taken as zero. The expressions for lift and moment then become

$$L_h^* = -\frac{2}{k^2}\left\{-\frac{k^2}{2} + ikC(k)\right\} \tag{17a}$$

$$L_\alpha^* = -\frac{2}{k^2}\left\{\frac{1}{2}(ik + k^2 a) + (1 + ik\lambda)C(k)\right\} \tag{17b}$$

$$M_h^* = -\frac{2}{k^2}\left\{\frac{ak^2}{2} - \left(\frac{1}{2} + a\right)ikC(k)\right\} \tag{17c}$$

$$M_{\alpha}^{*} = -\frac{2}{k^2}\left\{ \frac{1}{2}ik\lambda - \frac{1}{2}k^2\left(\frac{1}{8}+a^2\right) - \left(a+\frac{1}{2}\right)(1+ik\lambda)\,C(k)\right\} \qquad (17d)$$

Two subcases are now considered. Case 5a is based on the use of the values derived by Yates (Ref. 15) and given in Table 3b. The V-g curve corresponding to this calculation is shown in Figure 5a and gives a flutter velocity of approximately 30 knots with $g \cong 0$ and 32 knots with $g = 0.02$; this compares with a value of 38.5 knots with $g = 0.02$ as computed by Yates himself (Ref. 15). Note, however, that the slope of the critical V-g curve for small values of g is so flat that only very slight changes in the computed points would be necessary to alter the flutter velocity by a very large amount. Further, Yates employed six calculated modes (two bending and four torsion) in his analysis, while the present calculations are based on two measured mode shapes (one bending and one torsion)[*]. In any event, it would appear that the modified strip theory proposed by Yates (and as employed in this report) is overconservative by a significant degree.

Case 5b employs the measured steady state values shown in Table 3a. The corresponding V-g curve is shown in Figure 5b, giving a flutter speed of approximately 26 knots.

Case 6. Calculations Based on Appliedation of the Generalized Kutta Condition

As proposed a number of years ago (Refs. 4, 8, 9, 21), a generalization of the Kutta condition can be formulated in terms of a factor λ_1 applied to the

---

[*] Some rather simple considerations show that our use of the two measured modes should give reasonably accurate results. Any discrepancies arising between using calculated and measured modes is the result of differences in the dominant terms rather than in an insufficient number of modes.

trailing edge tangential velocity. Defining $\lambda_1 = 1 - \eta e^{i\phi_0}$, and replacing the constant factor $\frac{1}{2} + a$ by $a - x_{cp}$ as was done in Case 2, the lift and moment become:

$$L_h^* = - \frac{2}{k^2} \left\{ - \frac{k^2}{2} + ik\eta e^{i\phi_0} [C(k) + \sigma_h] \right\} \tag{18a}$$

$$L_\alpha^* = - \frac{2}{k^2} \left\{ \frac{1}{2}(ik + k^2 a) + \left[1 + ik\left(\frac{1}{2} - a\right)\right] \eta e^{i\phi_0} [C(k) + \sigma_\alpha] \right\} \tag{18b}$$

$$M_h^* = - \frac{2}{k^2} \left\{ \frac{ak^2}{2} - (a - x_{cp}) ik\eta e^{i\phi_0} [C(k) + \sigma_h] - \frac{i\lambda_1 k}{2} \right\} \tag{18c}$$

$$M_\alpha^* = - \frac{2}{k^2} \left\{ \frac{1}{2} ik \left(\frac{1}{2} - a\right) \eta e^{i\phi_0} - \frac{\lambda_1}{2} - \frac{1}{2} k^2 \left(\frac{1}{8} + a^2\right) \right.$$
$$\left. - (a - x_{cp}) \left[1 + ik\left(\frac{1}{2} - a\right)\right] \eta e^{i\phi_0} [C(k) + \sigma_\alpha] \right\} \tag{18d}$$

We shall assume that

$$\eta = \frac{\beta}{\cos \phi_0} \tag{19a}$$

$$\beta = \frac{C_{l_\alpha}}{2\pi} \tag{19b}$$

and shall select specific values of ϕ_0.

Case 6a is based on the use of measured data for C_{l_α} and x_{cp} (Table 3a), with V-g curves corresponding to several values of ϕ_0 shown in Figure 6a. The value of $\phi_0 = 0$ gives a flutter speed of approximately 42 knots, compared with the experimental value of 48.1 knots; a calculation using the value of

$\phi_0 = 10°$ gives almost exactly 48 knots. It is interesting to note that the Kutta condition modification alone (e.g., without any simultaneous correction on C_{l_α} and x_{cp}) gives computed results that agree with the actual hydrofoil flutter velocity of 48 knots only for the somewhat unrealistic value of $\phi_0 = -30°$ (Ref. 4). Even for $\phi_0 = 0$, however, these new results are quite good, especially if one considers that we have spoken only of the flutter speed corresponding to $g = 0$; in fact, based on zero forward speed tests, the damping of the SwRI flutter model was estimated to be $g_h = 0.016$ and $g_\alpha = 0.042$ in air, and is roughly estimated to be $g_h = 0.070$ and $g_\alpha = 0.047$ in water.† Calculated data corresponding to the critical frequency are also shown in Figure 6a, giving a value of $\frac{\omega}{\omega_\alpha} = 0.82$ and $\frac{1}{k} = 1.34$ (for $g = 0$) which are in good agreement with the measured values.

Case 6b is based on the use of Yates' calculated data for C_{l_α} and x_{cp} (Table 3b), with V-g curves corresponding to several values of ϕ_0 shown in Figure 6b, along with data for critical frequency. The value of $\phi_0 = 0$ (for $g = 0$) gives a flutter velocity of 45 knots and $\frac{\omega}{\omega_\alpha} = 0.82 \left(\frac{1}{k} = 1.43 \right)$. These are again in very good agreement with the measured values.

Finally, Case 6c differs from Case 6a only in that the center of pressure is taken as $x_{cp} = -\frac{1}{2}$, with the results being shown in Figure 6c. Again, with $\phi_0 = 0$ and $g = 0$, the flutter speed is 45 knots and $\frac{\omega}{\omega_\alpha} = 0.82$ $\left(\frac{1}{k} = 1.43 \right)$.

---

†Data provided by Mr. Guido E. Ransleben, Jr.

DISCUSSION AND CONCLUSIONS

A summary of the results obtained from the various cases studied is given in Table 4. Of these, only Case 6 gives results that are closely comparable with the measured values of flutter speed and frequency.

There can be no question that variations in lift curve slope and in center of pressure location, as suggested by Yates (Ref. 15), have a most profound influence on the flutter speed and frequency of a foil having parameters similar to those of the SwRI flutter model (Ref. 1). While the calculated results presented in this report are not in exceptionally close agreement with those of Yates, it appears this may be the consequence, in part, of sensitivity of the computations and also of the use of measured modes in the present calculations and of calculated modes by Yates. Both sets of results, however, lead one to conservative predictions, and this in itself is a significant advance over practically all previous results. The difference between using the measured values of lift curve slope and the center of pressure location and that of using those calculated from lifting surface theory is apparently not large. Yates' method certainly deserves further study and application.

The semiempirical method based on generalization of the Kutta conditions (Refs. 8, 9, 21), and used here in conjunction with lift curve slope and center of pressure modifications, appears to give excellent results (Case 6). As with the Yates' method, the differences arising from various C_{l_α} and x_{cp} values are small. An essential point is again that the flutter predictions are

conservative, as well as being in close agreement with the measured flutter speed and frequency. It would mean that the applications of the procedures employed in Case 6 to other hydrofoil flutter studies would be quite useful and revealing, if done prudently.

The role of structural damping could be significant; in Case 6, the inclusion of small positive damping would serve to improve even further the agreement between calculated and measured flutter speed.

ACKNOWLEDGEMENTS

REFERENCES

1. Abramson, H. N., and Ransleben, G. E., Jr., "An Experimental Investigation of Flutter of a Fully Submerged Subcavitating Hydrofoil," AIAA J. Aircraft, 2, 5, pp. 439-42, September-October 1965.

2. Abramson, H. N., Chu, W. H., and Irick, J. T., "Hydroelasticity - With Special Reference to Hydrofoil Craft," Rept. 2557, Naval Ship Research and Development Center, Washington, D. C. 20007, September 1967.

3. Chu, W. H., "A Critical Re-Evaluation of Hydrodynamic Theories and Experiments in Subcavitating Hydrofoil Flutter," J. Ship Res., 10, 2, pp. 122-32, June 1966.

4. Abramson, H. N., and Langner, C. G., "Correlation of Varicus Subcavitating Hydrofoil Flutter Predictions Using Modified Oscillatory Lift and Moment Coefficients," Final Rept., Contract No. NObs-88599, Southwest Research Institute, June 1964.

5. Abramson, H. N., and Chu, W. H., "A Discussion of the Flutter of Submerged Hydrofoils," J. Ship Res., 3, 4, pp. 9-12, March 1960.

6. Ransleben, G. E., Jr., and Abramson, H. N., "Experimental Determination of Oscillatory Lift and Moment Distributions of Fully Submerged Flexible Hydrofoils," J. Ship Res., 7, 2, pp. 24-41, October 1963.

7. Abramson, H. N., and Ransleben, G. E., Jr., "Experimental Unsteady Airfoil Lift and Moment Coefficients for Low Values of Reduced Velocity," AIAA J., 1, 6, pp. 1441-3, June 1963.

8. Chu, W. H., and Abramson, H. N., "An Alternative Formulation of the Problem of Flutter in Real Fluids," J. Aerospace Sci., 26, 10, pp. 683-4, October 1959.

9. Chu, W. H., "Three-Dimensional Effect of Flutter in a Real Fluid," J. Aerospace Sci., 29, 3, pp. 374-5, March 1962.

10. Yates, E. C., Jr., "Calculation of Flutter Characteristics for Finite-Span Swept or Unswept Wings at Subsonic and Supersonic Speeds by a Modified Strip Analysis," NACA RML57L10, March 1958.

11. Yates, E. C., Jr., "Some Effects of Variations in Density and Aerodynamic Parameters on the Calculated Characteristics of Finite Span Swept and Unswept Wings at Subsonic and Supersonic Speeds," NACA TM X-182, Declassified, February 6, 1962.

12. Yates, E. C., Jr., "Use of Experimental Steady-Flow Aerodynamic Parameters in the Calculation of Flutter Characteristics for Finite-Span Swept or Unswept Wings at Subsonic, Transonic and Supersonic Speeds," NACA TM X-183, Declassified, February 8, 1963.

13. Yates, E. C., Jr., "Subsonic and Supersonic Flutter Analysis of a Highly Tapered Swept-Wing Planform, Including Effects of Density Variation and Finite Wing Thickness, and Comparison with Experiments," NACA X-764, March 1963, Declassified, October 12, 1966.

14. Yates, E. C., Jr., "Modified-Strip-Analysis Method for Predicting Wing Flutter at Subsonic to Hypersonic Speeds," AIAA J. Aircraft, 3, 1, pp. 25-9, January-February 1966.

15. Yates, E. C., Jr., "Flutter Prediction at Low Mass-Density Ratios with Application to the Finite-Span Subcavitating Hydrofoil," AIAA Paper No. 68-472.

16. Widnall, S. E., "Unsteady Loads on Hydrofoils Including Free Surface Effects and Cavitation," MIT Fluid Dynamics Research Lab. Rept. No. 64-2, Mass. Inst. of Tech., Contract Nonr 1841(81), June 1964.

17. Ashley, H., Widnall, S. E., and Landahl, M. T., "New Directions in Lifting Surface Theory," AIAA J., 3, 1, pp. 3-16, January 1965.

18. Rowe, W. S., "Collocation Method for Calculating the Aerodynamic Pressure Distributions on a Lifting Surface Oscillating in Subsonic Compressible Flow," AIAA Symp. Struct. Dyn. and Aeroelasticity, Boston, August 30, 1965.

19. Scanlan, R. H., and Rosenbaum, R., Aircraft Vibration and Flutter, The MacMillan Co., New York, 1951.

20. Reissner, E., and Stevens, J. E., "Effects of Finite Span on the Airload Distributions for Oscillating Wings, II, Methods of Calculation and Examples of Application," NACA TN 1195, October 1947.

21. Chu, W. H., "Some Contributions to Unsteady Hydrodynamics in Engineering," Ph. D. Dissertation, The Johns Hopkins University (University Microfilm, Inc.), June 1963.

TABLES

TABLE 1a. SwRI FLUTTER MODEL-MEASURED MODE SHAPES (IN AIR)

	y/L	f_h	f_α
Root	0	0	0
	0.1	0.025	0.213
	0.2	0.075	0.403
	0.3	0.158	0.570
	0.4	0.260	0.710
	0.5	0.370	0.825
	0.6	0.488	0.915
	0.7	0.610	0.966
	0.8	0.738	0.990
	0.9	0.870	0.999
Tip	1.0	1.000	1.000

TABLE 1b. SwRI FLUTTER MODEL CHARACTERISTICS (Ref. 1)

Model Parameter	Measured Value
Aspect ratio	5.00
Semichord	0.50 ft
Elastic axis location, a	-0.50
Center of gravity location, x_α (from a.e.)	0.524
Radius of gyration, r_α^2	0.512
Bending stiffness*, EI	3.40×10^6 lb-in^2
Torsional stiffness*, GJ	0.973×10^6 lb-in^2
Frequency ratio, ω_h/ω_α	0.490
Torsional frequency, ω_α	20.5 cps
Total weight (wing only)	121.2 lb

*The calculated ω_α and ω_h are about 18.7 cps and 11.2 cps. The calculated ω_h/ω_α is about 0.60. However, the measured frequencies are possibly more accurate than the calculated values based on measured EI and GJ.

TABLE 2. VALUES OF OSCILLATORY CENTER OF PRESSURE
LOCATIONS IN SEMICHORDS (Ref. 6)

$1/k$	$\bar{k}$	$2\tilde{x}_{cp_h}$	$2\tilde{x}_{cp_\alpha}$	$x^*_{cp_h}$	$x^*_{cp_\alpha}$
0.4	2.5	0.85	0.92	0.54	0.61
0.8	1.25	0.630	0.76	0.32	0.45
1.0	1.00	0.560	0.70	0.25	0.39
1.5	0.667	0.480	0.58	0.17	0.27
1.8	0.556	0.48	0.53	0.17	0.22
2.0	0.5	0.48	0.50	0.17	0.19

TABLE 3a. EXPERIMENTAL VALUES FOR STEADY
STATE LIFT CURVE SLOPE C_{l_a} AND CENTER OF
PRESSURE, x_{cp} (Ref. 6)

y^*	C_{l_a} (rad.)	x_{cp} (semichords)
0	4.01	-0.522
0.1	4.01	-0.522
0.2	4.01	-0.536
0.3	4.01	-0.540
0.4	4.01	-0.544
0.5	4.01	-0.552
0.6	4.01	-0.564
0.7	3.724	-0.580
0.8	3.438	-0.596
0.9	2.58	-0.632
1.0	0	-0.640

TABLE 3b. THEORETICAL VALUES FOR STEADY
STATE LIFT CURVE SLOPE C_{l_a} AND CENTER OF
PRESSURE x_{cp} FOR THE SwRI FLUTTER MODEL

y*	C_{l_a} (rad.)	x_{cp} (semichords)
0	4.85	-0.531
0.1	4.84	-0.531
0.2	4.80	-0.533
0.3	4.71	-0.535
0.4	4.60	-0.536
0.5	4.44	-0.542
0.6	4.21	-0.550
0.7	3.86	-0.563
0.8	3.36	-0.578
0.9	2.54	-0.605
1.0	0	-0.646

TABLE 4. SUMMARY OF RESULTS

Case No.	Brief Description	Flutter Speed, V_f (knots)	Critical Frequency, ω/ω_α
Experiment	SwRI flutter model (b = 0.5 ft, ω_α = 20.5 cps)	48.1	0.854
1	Reissner-Stevens Theory	108	--
2	Exp. (steady) x_{cp}, $C_{l_\alpha} = 2\pi$; $\sigma_j \cdot j = \alpha$, h	67	--
3	Unsteady $x^*_{cp_\alpha}$, $x^*_{cp_h}$	70	--
4a	Exp. (steady) x_{cp}, C_{l_α}; $\sigma_j = 0$	100	--
4b	Exp. (steady) x_{cp}, C_{l_j}; σ_j	76	--
5a	Yates Theory; Yates x_{cp}, C_{l_α}	30	--
5b	Yates Theory; exp. (steady) x_{cp}, C_{l_α}	26	--
6a	Exp. (steady) x_{cp}, C_{l_α}; Generalized Kutta condition, $\phi_0 = 0$	42	0.82
6b	Yates x_{cp}, C_{l_α}; Generalized Kutta condition, $\phi_0 = 0$	45	0.82
6c	Exp. (steady) C_{l_α} ; $x_{cp} = -1/2$ Generalized Kutta condition, $\phi_0 = 0$	45	0.82

ILLUSTRATIONS
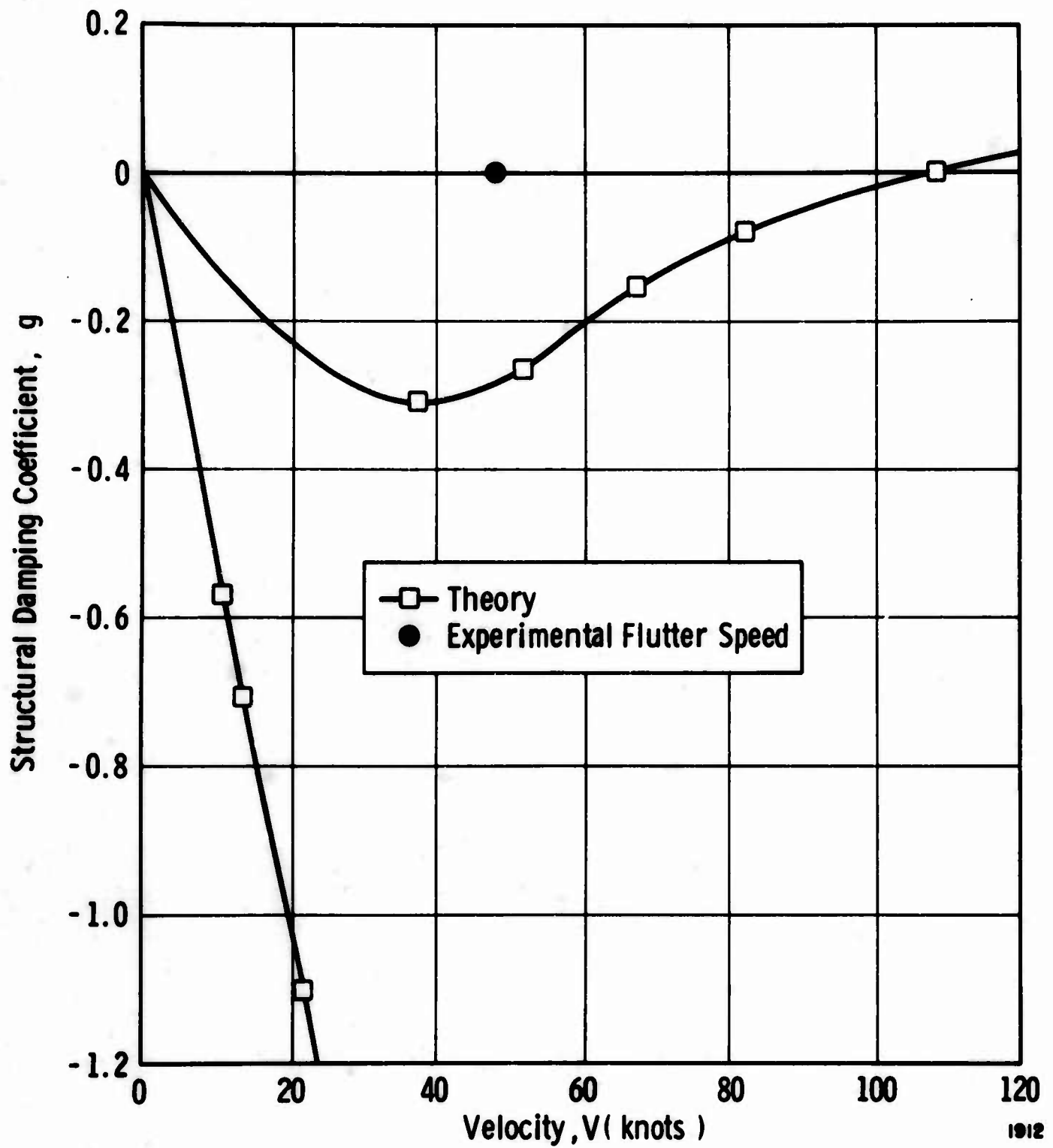
Figure 1. V-g Curves For Case 1 (Reissner - Stevens Theory)
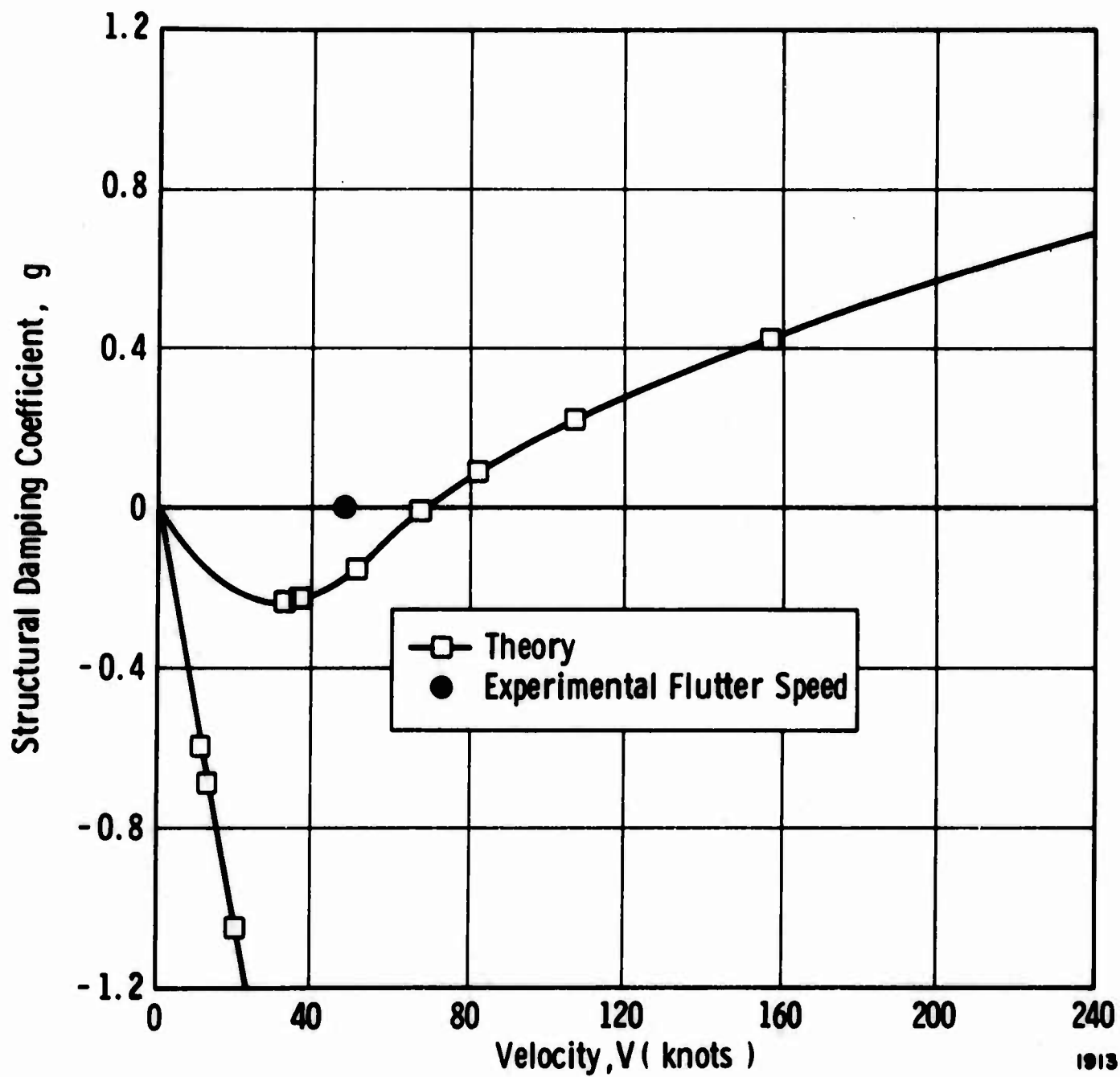
Figure 2. V-g Curves For Case 2

1913
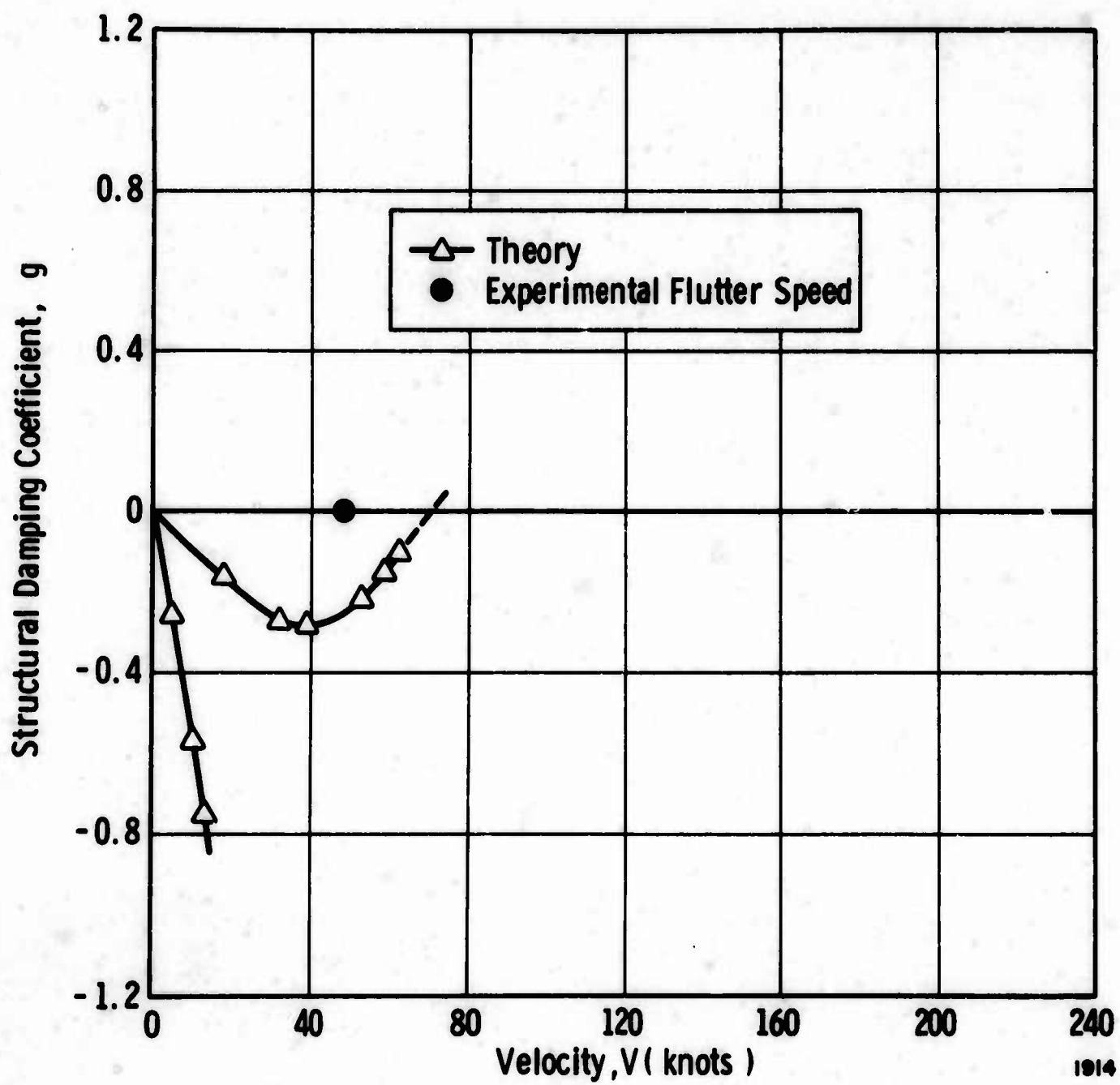
Figure 3. V-g Curves For Case 3
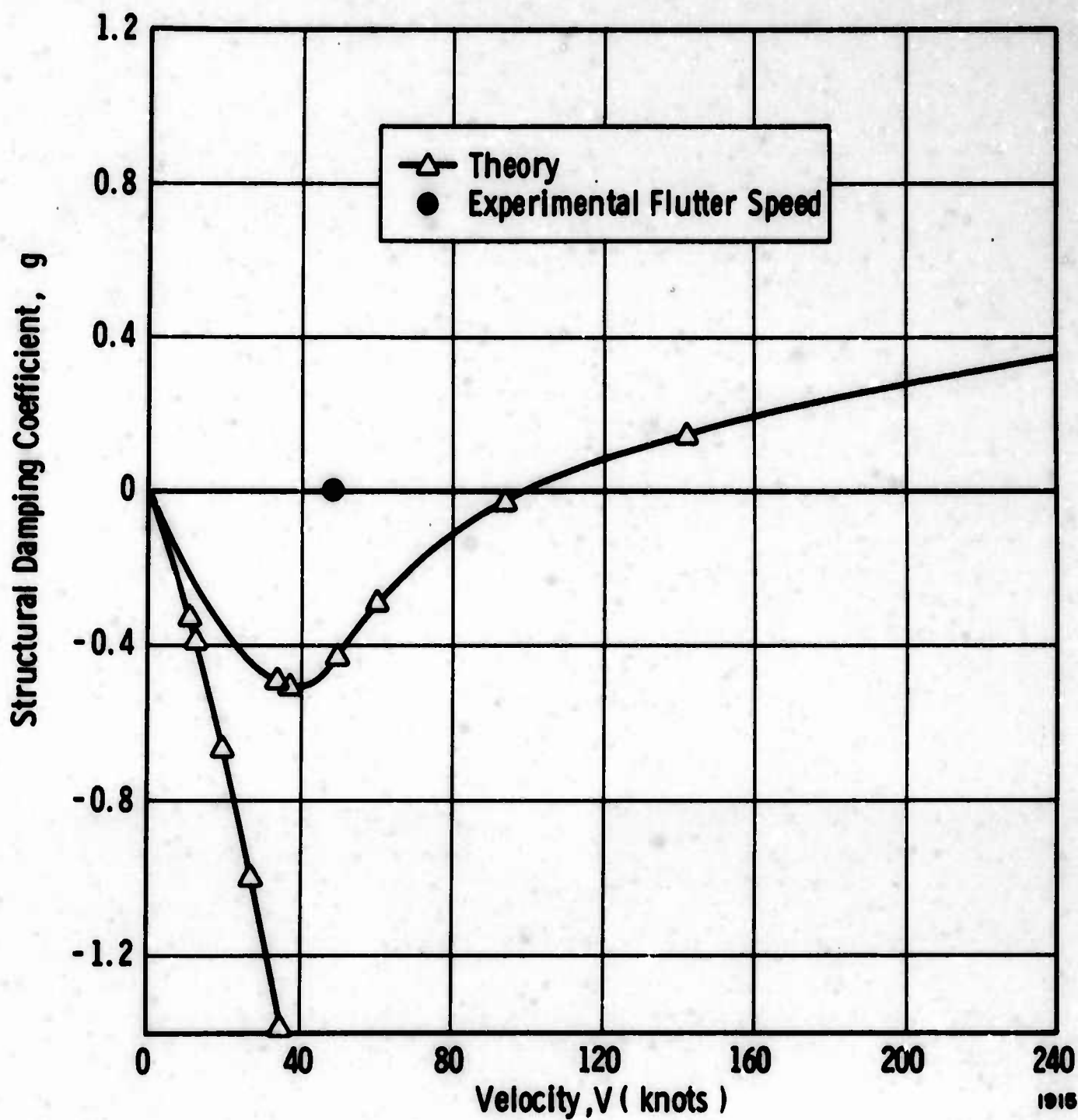
Figure 4a. V-g Curves For Case 4a

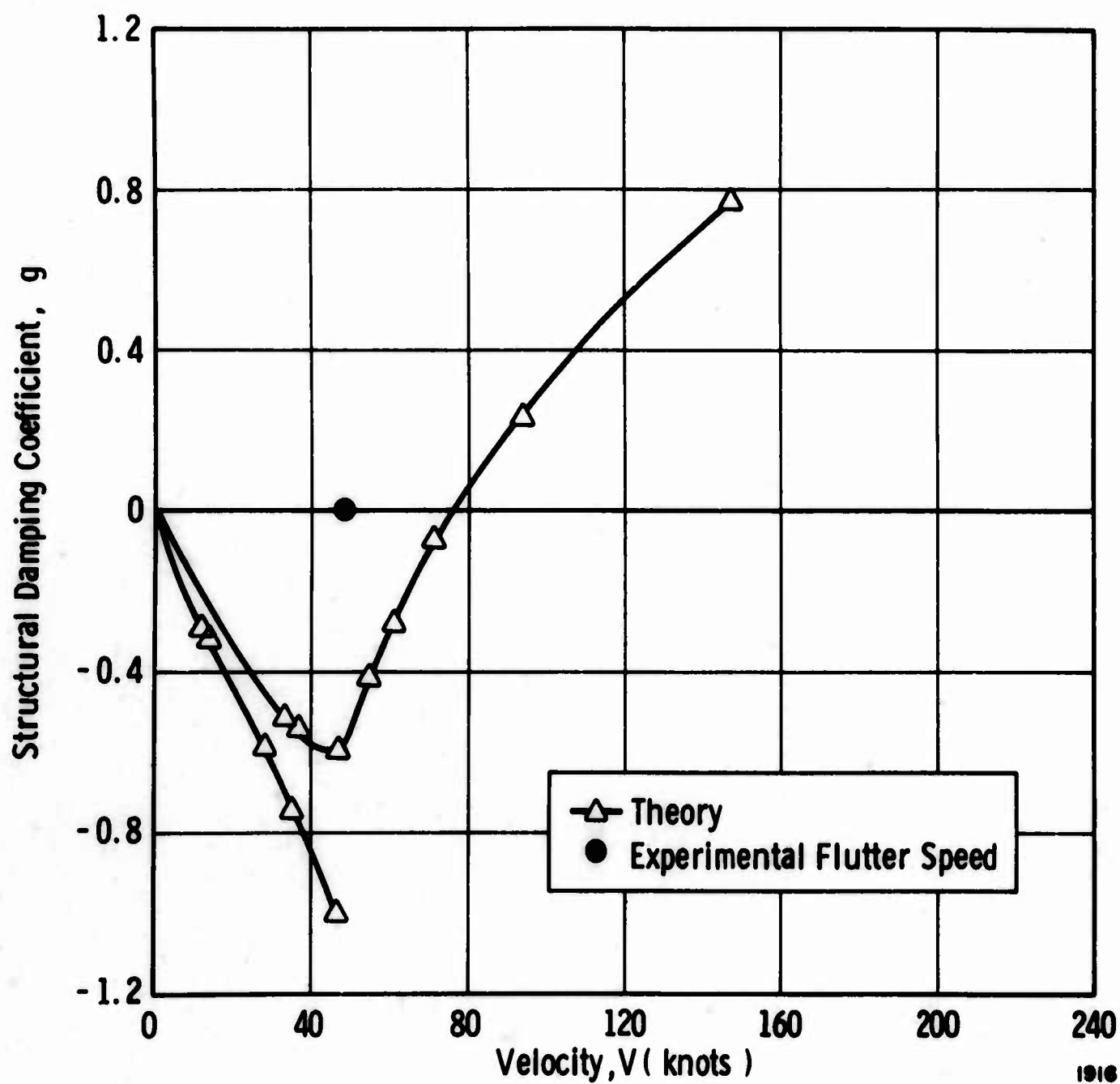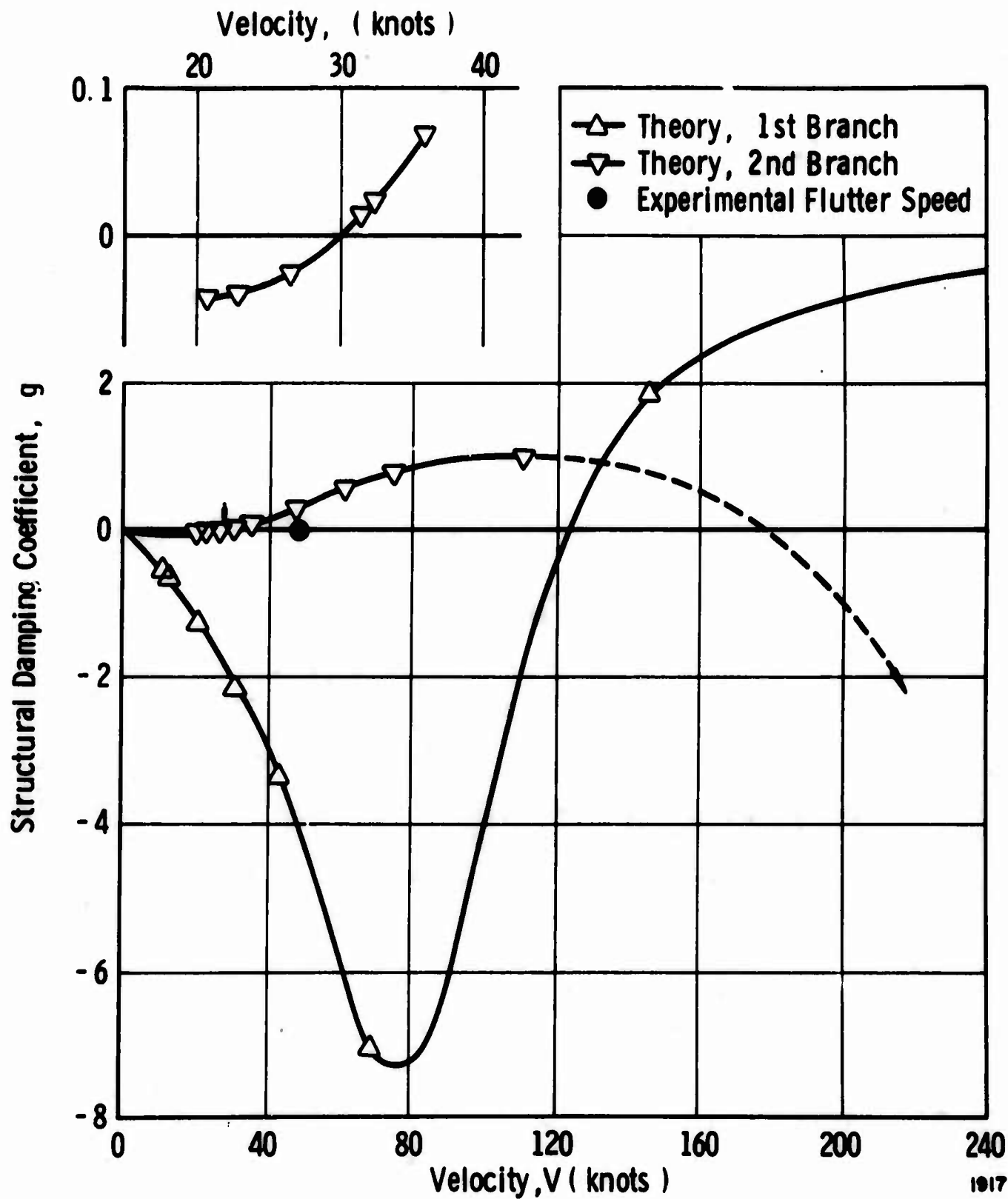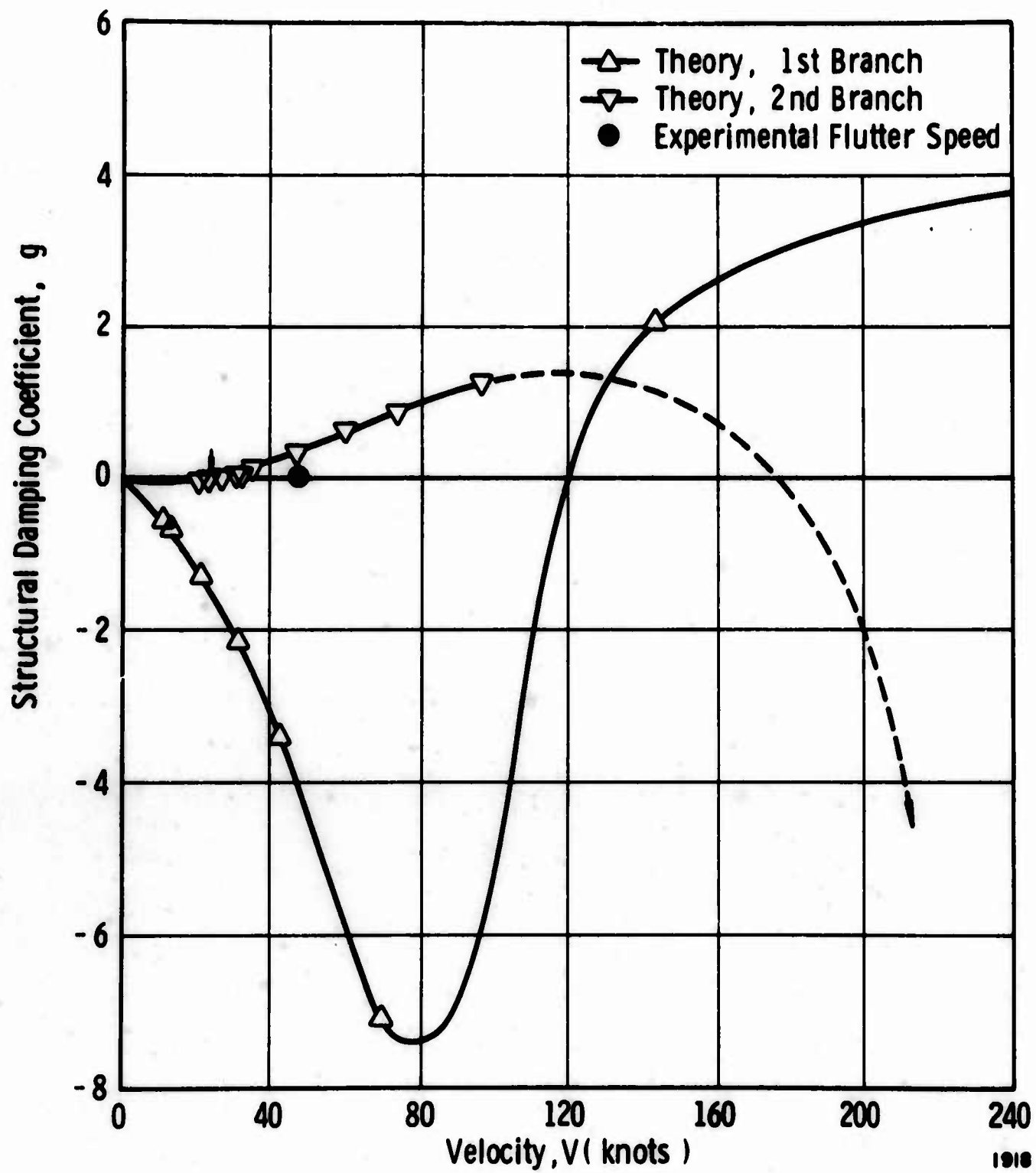Figure 4b. V-g Curves For Case 4b

Figure 5a. V-g Curves For Case 5a

Figure 5b. V-g Curves For Case 5b

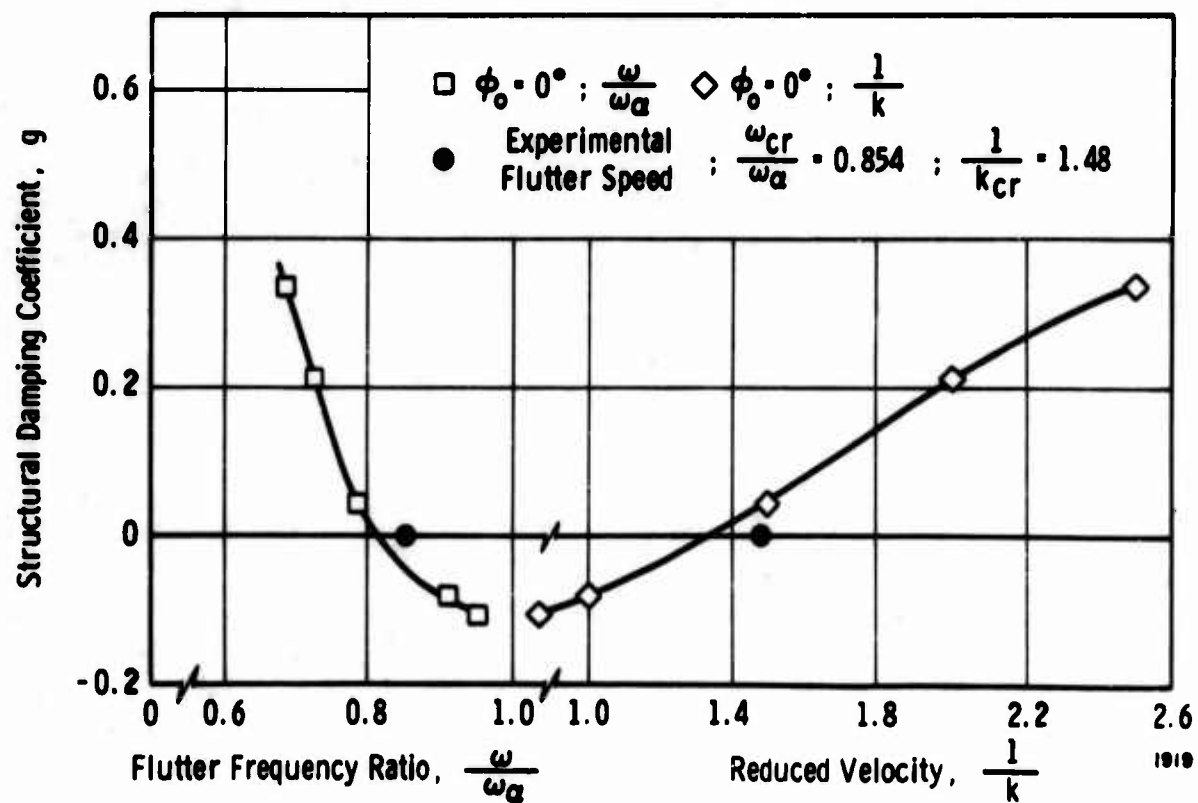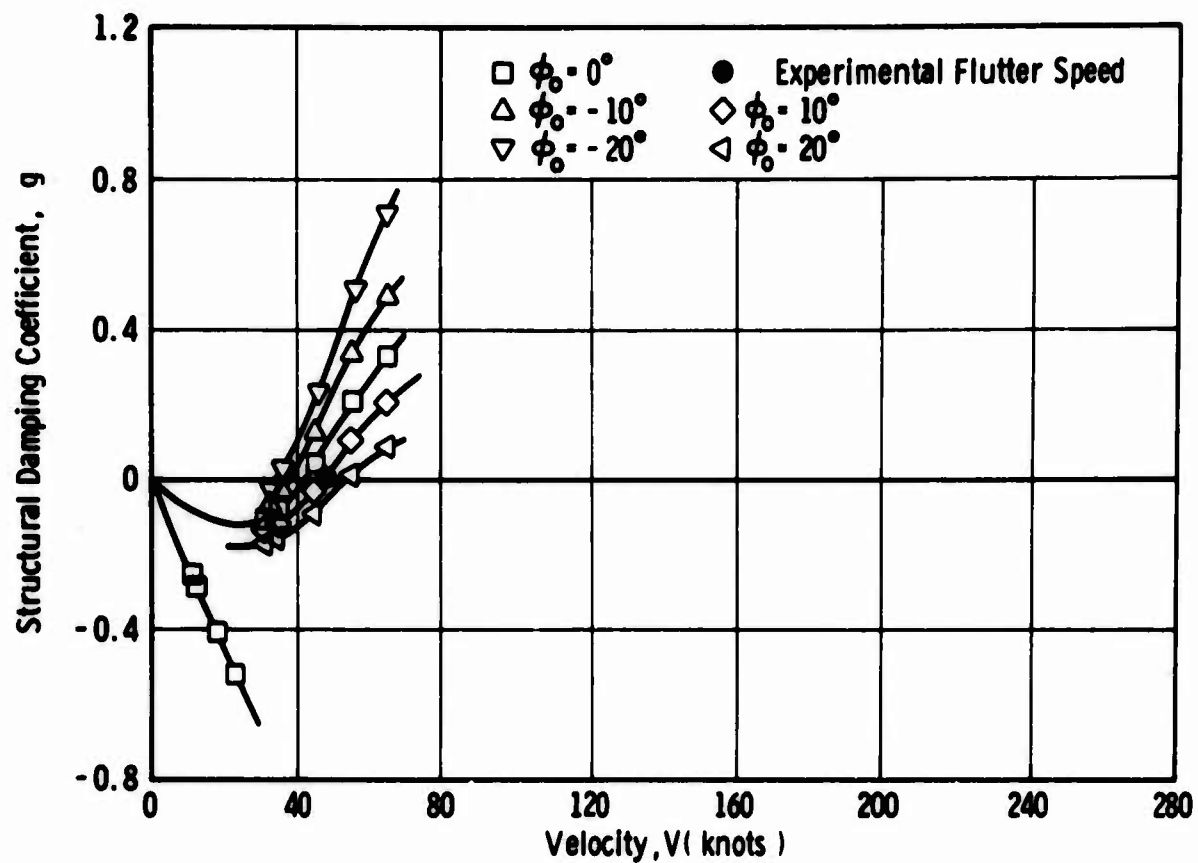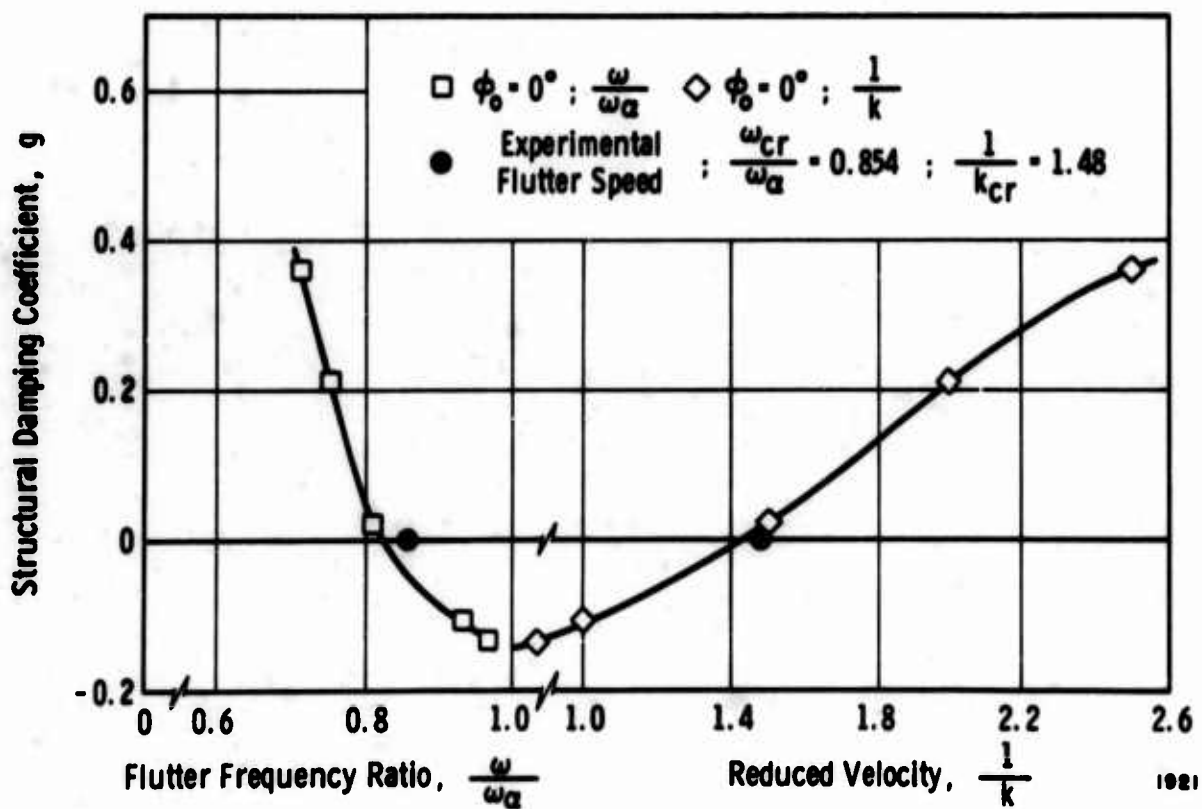Figure 6a. V-g Curves, $\frac{\omega_{cr}}{\omega_\alpha}$ And $\frac{1}{k_{cr}}$ Curve For Case 6a
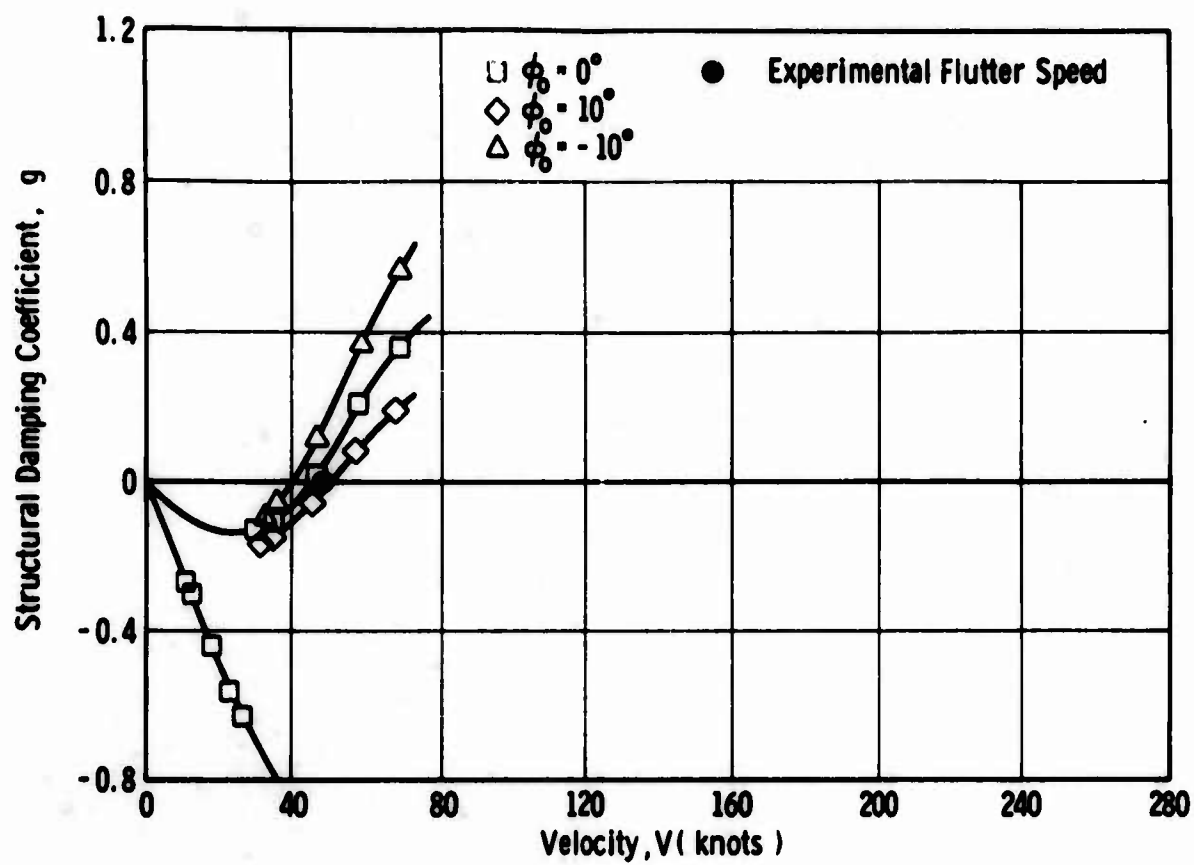
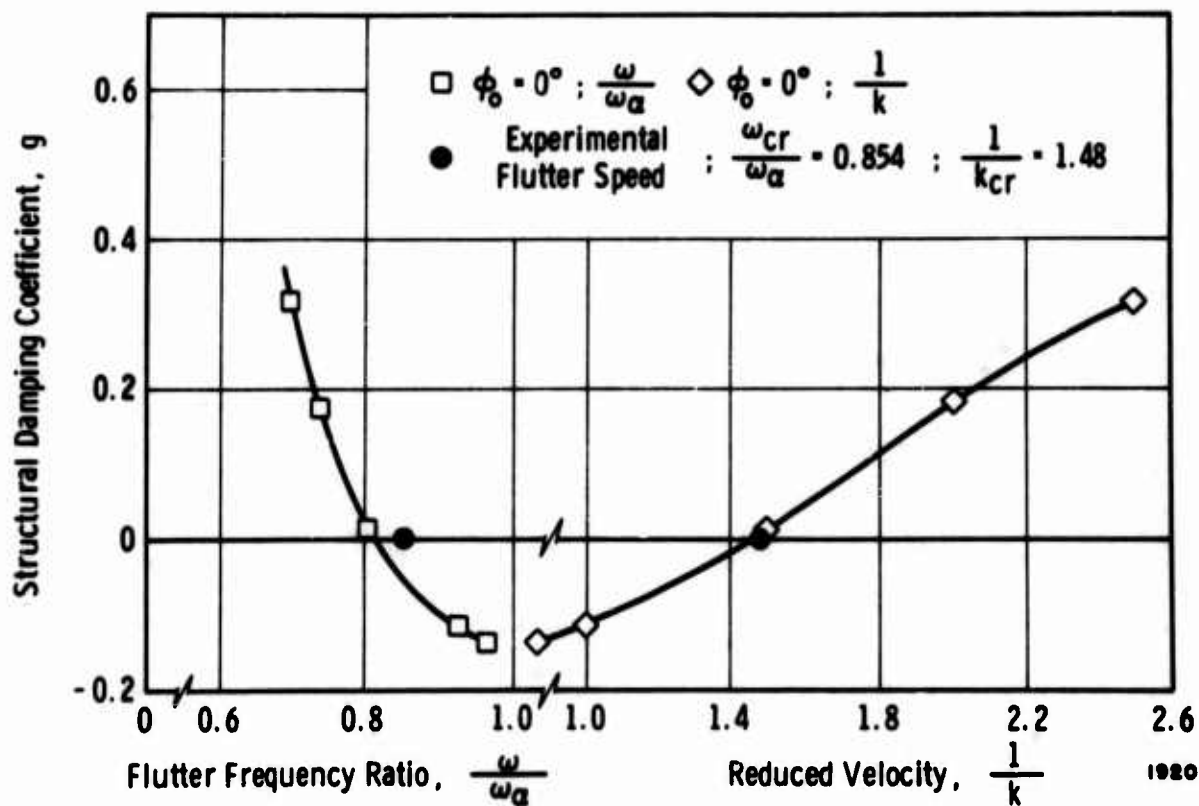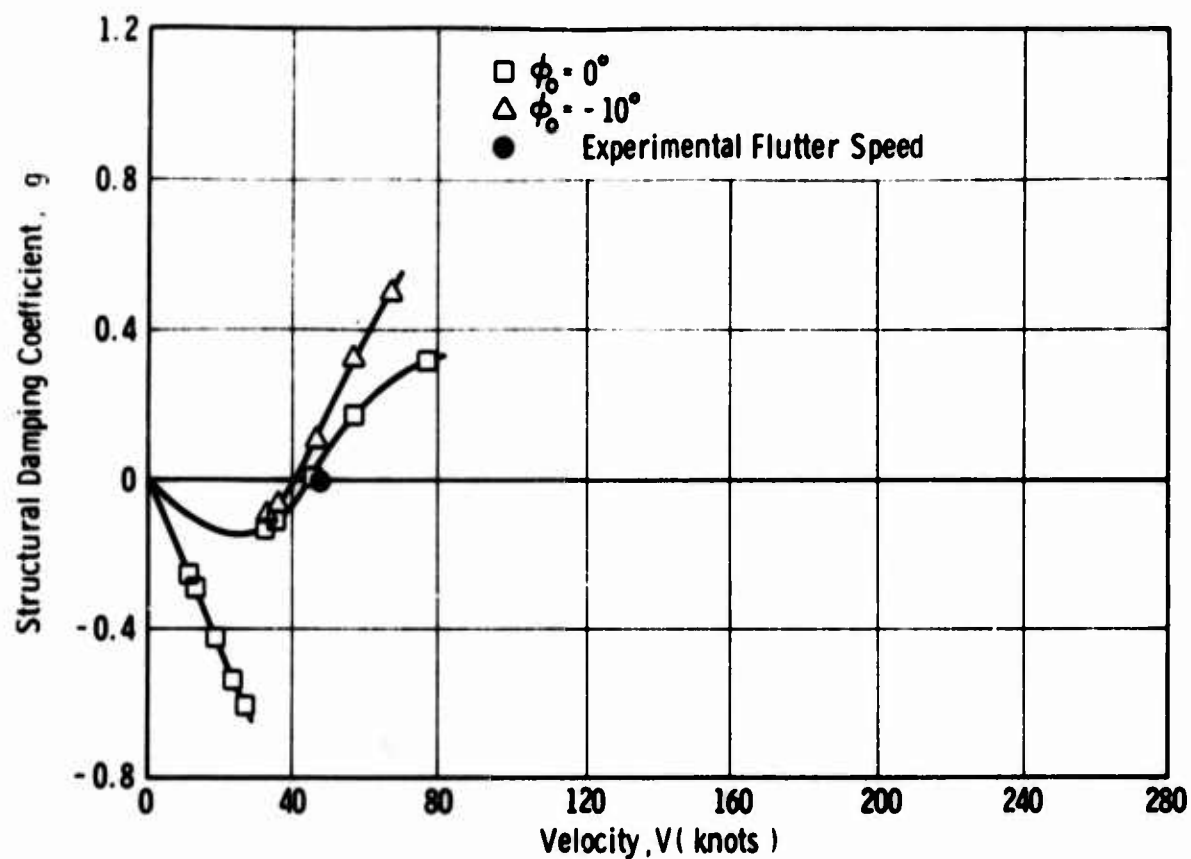Figure 6b. V-g Curves, $\dfrac{\omega_{cr}}{\omega_\alpha}$ And $\dfrac{1}{k_{cr}}$ Curve For Case 6b

Figure 6c. V-g Curves, $\frac{\omega_{cr}}{\omega_\alpha}$ And $\frac{1}{k_{cr}}$ Curve For Case 6c

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Southwest Research Institute 8500 Culebra Road San Antonio, Texas 78228	Unclassified
	2b. GROUP

3. REPORT TITLE

Further Calculations of the Flutter Speed of a Fully Submerged Subcavitating Hydrofoil

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report (1 February 1968 — 31 July 1968)

5. AUTHOR(S) (First name, middle initial, last name)

Wen-Hwa Chu

H. Norman Abramson

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1968	34	21

8a. CONTRACT OR GRANT NO	9a. ORIGINATOR'S REPORT NUMBER(S)
N00014-68-C-0259	Technical Report dated June 1968 on
b. PROJECT NO	SwRI Project 02-2311
SF 013 02 01	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of the Head, Hydromechanics Laboratory, Naval Ship Research and Development Center.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Ship Research and Development Center Washington, D. C. 20007

13. ABSTRACT

Some new calculations of the flutter speed of the SwRI fully submerged subcavitating hydrofoil model are presented. Variations in lift curve slope and in center of pressure location are found to have a most profound influence on both flutter speed and frequency. When variations in these parameters are combined with a relaxation of the Kutta condition (proposed previously), excellent agreement with the measured flutter speed is obtained.

DD FORM 1473 (1 NOV 65)

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Hydrofoils Flutter Hydroelasticity						